

Stability properties of trapped Bose-Fermi gases mixture

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The stability of Bose-Fermi gases trapped in an isotropic potentials at ultracold temperature is strongly influenced by the interaction between the fermions and the bosons. At zero temperature, the stability criterion is given in this paper using variation method, the results show that whether a fermion-boson mixture is stable depends mainly on the interaction between the fermions and the bosons. For finite temperature, however, the stability is not only related to the coupling constants, but also to the temperature. The stability conditions for finite temperature are also derived and discuss in details in this paper.

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Since the realization of dilute alkali atomic vapor condensates (Bose-Einstein condensation or BEC) in 1995[1], large efforts have been made to study many-body effects and macroscopic properties of the gases, which may be more transparently demonstrated in BEC than in other many-body systems. For fermionic atomic vapor, however, it is difficult to achieve a degenerate gas. Since the evaporative cooling of a pure fermionic gas is ineffective at temperature sufficiently low due to the suppression of s -wave scattering between identical fermions. As theory and experiment advanced, a new rich phenomenology has appeared in which new conditions arise, which are not accessible in other BEC systems. One of the most stunning of these is the recent experimental demonstration of a condensate mixture composed of two spin states of ^{87}Rb [2]. The realization of two condensates mixture is related to the sympathetic cooling mechanism, i.e., the exchange of energy due to elastic collisions between atoms of cooled and thermal samples. Most recently, B.DeMarco and D.S.Jin [3] report their observation of degenerate Fermi gas using an evaporative cooling strategy. Although the strategy uses a two-component Fermi gas, the mixture of Bose and Fermi gas attracts a lot of attention from the viewpoint of both experiment and theoretical study.

The mixed system of Bose and Fermi particles is itself an interesting subject for investigation. The hydrogen deuterium system has been studied at the early stage of these investigations[4], and there is now a lot of literature devoted to the properties of pure degenerate trapped atomic Fermi gases[5-9].

In a recent paper, Mϕlmer has used a simple mean field models to study the spatial distribution of a Bose-Fermi gas mixture at $T = 0K$ within Thomas-Fermi approximation. The results show that the distributions depend strongly on the relative sign and magnitude of the boson-boson and boson-fermion scattering lengths. Here, we shall study the Bose-Fermi gas mixture using a variation method at zero temperature, this method was first introduced in[10] to study the BEC ground state in a harmonic trap of a Bose system, and later generalized by H.Shi and W.M.Zheng to study BEC with attractive interactions[11]. In addition, we study the stability of the Bose-Fermi gas mixture at finite temperature. The results show that there is a region of temperature in which the phase separation of the mixture happens. And the span of the region depends on the coupling constants.

To begin, we consider a second-quantized grand canonical Hamiltonian of interacting Bose and Fermi gases

$$\begin{aligned} H &= H_b + H_f + V_{bf}, \\ H_b &= \int dr \phi^\dagger(r) \left(\frac{p^2}{2m_b} - \mu_b + \frac{1}{2} m_b \omega_b r^2 \right) \phi(r) + \frac{g_{bb}}{2} \int \int dr dr' \phi^\dagger(r) \phi^\dagger(r') \phi(r') \phi(r), \\ H_f &= \int dr \psi^\dagger(r) \left(\frac{p^2}{2m_f} - \mu_f + \frac{1}{2} m_f \omega_f r^2 \right) \psi(r), \\ V_{bf} &= g_{bf} \int dr dr' \phi^\dagger(r) \psi^\dagger(r') \delta(r - r') \psi(r') \phi(r), \end{aligned} \quad (1)$$

where $\phi(r)$ and $\psi(r)$ denote boson and fermion field operators with masses m_b and m_f , respectively. For weakly interacting dilute gases, the interactions between the bosonic atoms are modeled by δ potentials and the interactions among the fermionic atoms are neglected, since the interactions between atoms at very low temperature is suppressed for polarized systems. g_{bb} and g_{bf} stand for boson-boson and boson-fermion coupling constant, respectively.

$$g_{bb} = \frac{4\pi\hbar^2}{m_b} a_{bb}, g_{bf} = \frac{2\pi\hbar^2}{m_{bf}} a_{bf},$$

$a_{bb}(a_{bf})$ are s -wave scattering length between boson and boson (boson and fermion), and m_{bf} is a reduced mass of the boson and the fermion. The chemical potentials μ_b and μ_f are determined through the conditions

$$N_b = \langle \int dr \phi^\dagger(r) \phi(r) \rangle, N_f = \langle \int dr \psi^\dagger(r) \psi(r) \rangle. \quad (2)$$

At $T = 0$, self-consistent mean field theory, assuming that all N bosonic particles in a gas populated the same state denoted by single particle wave function $\Phi(r)$, lead to a nonlinear Schrödinger equation (or the Gross-Pitaevskii equation) for $\Phi(r) = \langle \phi(r) \rangle$

$$[-\frac{\hbar^2}{2m_b} \nabla^2 + \frac{1}{2} m_b \omega_b^2 r^2 + g_{bb} n_b(r)] \Phi(r) = E_b \Phi(r), \quad (3)$$

we here omit quantities $g_{bf} n_f(r)$, which is smaller than $g_{bb} n_b(r)$ in the case of $N_b \gg N_f$. In order to get a degenerate fermionic gas, the boson particles appear in the system only as a coolant, so the number of bosons is always much larger than the number of fermions. In the same approximation, the fermionic wave function is given by a Slater determinant

$$\Psi(r_1, r_2, \dots, r_{N_f}) = \frac{1}{\sqrt{N_f!}} \begin{vmatrix} \Psi_1(r_1) & \Psi_1(r_2) & \cdots & \Psi_1(r_{N_f}) \\ \Psi_2(r_1) & \Psi_2(r_2) & \cdots & \Psi_2(r_{N_f}) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{N_f}(r_1) & \Psi_{N_f}(r_2) & \cdots & \Psi_{N_f}(r_{N_f}) \end{vmatrix}, \quad (4)$$

where $\Psi_i(r)$ is the single particle states determined by Hartree-Fock self-consistent equation

$$[-\frac{\hbar^2}{2m_f} \nabla^2 + \frac{1}{2} m_f \omega_f^2 r^2 + g_{bf} n_b(r)] \Psi_i(r) = E_i \Psi_i(r). \quad (5)$$

The density of the fermions is given by

$$n_f(r) = |\Psi(r)|^2. \quad (6)$$

In the semiclassical (Thomas-Fermi) approximation, the particle are assigned classical position and momenta, but the effects of quantum statistics are taken into account. Under this approximation, the Eqs.(3) and (5) for the boson and fermion wave function are equivalent to [13,14]

$$\begin{aligned} \frac{1}{2} m_b \omega_b^2 r^2 + g_{bb} n_b(r) &= \mu_b, \\ \frac{\hbar^2}{2m_f} [6\pi^2 n_f(r)]^{\frac{2}{3}} + \frac{1}{2} m_f \omega_f^2 r^2 + g_{bf} n_b(r) &= e_F. \end{aligned} \quad (7)$$

The main conclusion of this equations is discussed in Ref.[13]. We obtain $n_b(r) = \frac{1}{g_{bb}}(\mu_b - \frac{1}{2} m_b \omega_b^2 r^2)$ from the first line of Eqs(7). Substituting $n_b(r)$ into the second line of Eqs(7), we yield

$$\frac{\hbar^2}{2m_f} [6\pi^2 n_f(r)]^{\frac{2}{3}} + \frac{1}{2} m_f \omega_f^2 r^2 + \frac{g_{bf}}{g_{bb}} (\mu_b - \frac{1}{2} m_b \omega_b^2 r^2) = e_F, \quad (8)$$

this equation shows that the fermions experience a potential minimum in the center of the trap if $g_{bf}/g_{bb} < m_f \omega_f^2 / m_b \omega_b^2$, in this case the entire distribution behaves like a fermionic core within the Bose condensate. The fermion density is a constant throughout the Bose condensate if $g_{bf}/g_{bb} = m_f \omega_f^2 / m_b \omega_b^2$. Whereas the fermions are repelled from the center of the trap and localized near the edge of the Bose condensate if $g_{bf}/g_{bb} > m_f \omega_f^2 / m_b \omega_b^2$, i.e. a phase separation occurs in this system. We would like to note that the distribution of BEC remains unchanged in the above discussions, since we assume $N_b \gg N_f$. To drive Eq.(8), we assume that the Thomas-Fermi approximation(TFA) is valid. The coupling constant g_{bb} and g_{bf} may take any value as long as the TFA is available, and the phase separation depend mainly on ratio g_{bf}/g_{bb} . In what follows we discuss the separation of the bosonic and fermionic parts from the other aspect for zero temperature by using variation method, the results are indeed

different from those under TFA. We note the solution of Eq.(5) requires prior knowledge of the boson density profile $n_b = |\Phi(r)|^2$. To obtain the density profile, we have to solve the Gross-Pitaevskii equation (3). There are a large number of literatures devoted to solve the Gross-Pitaevskii equation[12], we here use a variation method[11] to solve the problem. For a isotropic trapping potential, we may assume the trial wave function for $\Phi(r)$ in Eq.(3) to be

$$\Phi(r) = \sqrt{N_b} \omega^{\frac{3}{4}} \left(\frac{m_b}{\pi \hbar} \right)^{\frac{3}{4}} e^{-m_b \omega r^2 / 2 \hbar}, \quad (9)$$

where ω is the effective frequency and is taken as a variational parameter. Substituting Eq.(9) into Eq.(3), we obtained the ground-state energy

$$E_b[\Phi] = E_b(\omega) = \frac{3}{4} N_b \hbar \omega + \frac{3}{4} N_b \hbar \frac{\omega_b^2}{\omega} + g_{bb} N_b^2 \left(\frac{\omega m_b}{2\pi \hbar} \right)^{\frac{3}{2}}. \quad (10)$$

If $E_b(\omega)$ is plotted as a function of ω , one sees that a stable local minimum exists only up to a certain maximum number of atoms for $g_{bb} < 0$ [11]. The critical point occurs where

$$\left. \frac{\partial E_b(\omega)}{\partial \omega} \right|_{(\omega=\omega_c, N_b=N_{bc})} = 0, \quad \text{and} \quad \left. \frac{\partial^2 E_b(\omega)}{\partial \omega^2} \right|_{\omega=\omega_c, N_b=N_{bc}} > 0. \quad (11)$$

Here, ω_c stands for the variational parameter that minimizes the ground state energy. Using equation (10), for $g_{bb} < 0$ the critical number of bosons is given by

$$N_b^c < 2 \hbar \omega_b^2 (\omega_c)^{-\frac{5}{2}} \frac{1}{|g_{bb}|} \left(\frac{2\pi \hbar}{m_b} \right)^{\frac{3}{2}}. \quad (12)$$

where ω_c satisfies

$$\hbar \omega_c^2 - \hbar \omega_b^2 + 2 g_{bb} N_b \left(\frac{m_b}{2\pi \hbar} \right)^{\frac{3}{2}} \omega_c^{\frac{5}{2}} = 0. \quad (13)$$

Parameter $\omega_b \sim 166 \text{ Hz}$ relevant to the experiment gives $N_b^c \sim 1400$, which is in good agreement with the experiment[1,11]. The solution ω_c of Eq.(13) against g_{bb} is plotted in Fig.1, which shows that as $|g_{bb}|$ increases, the variation parameter ω_c decreases, and it has a maximum equal to ω_b at $g_{bb} = 0$. We will use this solution to study the stability of the mixture at zero temperature below.

We may determine the ground-state energy functional of the fermions provided $n_b(r)$ is known. In terms of the fermion distribution $n_f(r)$, the energy functional E_f of the fermions is given by[15]

$$E_f = E_f[n_f(r)] = \int \frac{d^3 r}{6\pi^2} \frac{\hbar^2}{2m_f} [6\pi^2 n_f(r)]^{5/3} + \frac{1}{2} \int d^3 r m_f \omega_f^2 r^2 n_f(r) + \int d^3 r g_{bf} n_f(r) n_b(r), \quad (14)$$

since the interaction between the bosons and the fermions is rather weak, we may consider a Gaussian function as a trial fermions' distribution

$$n_f(r) = N_f \Omega^{3/2} \left(\frac{m_f}{\pi \hbar} \right)^{3/2} e^{-\frac{m_f \Omega (r-r_f)^2}{\hbar}}. \quad (15)$$

Here r_f and Ω is treated as variation parameters. Substituting this wave functions into Eq.(14), one obtains

$$E_f = E_f(\Omega, r_f, N_f, \omega_c) = P + \frac{1}{2} m_f \omega_f^2 r_f^2 N_f + \hbar N_b N_f (\pi G)^{\frac{3}{2}} e^{-G r_f^2} \quad (16)$$

with

$$P = P(\Omega, N_f) = \left(\frac{3}{5} \right)^{\frac{3}{2}} \frac{1}{\pi} (6\pi^2)^{\frac{2}{3}} \hbar \Omega N_f^{\frac{5}{3}} + \frac{3\hbar \omega_f^2}{4\Omega} N_f$$

$$G = G(\Omega, \omega_c) = \frac{m_f m_b \omega_c \Omega}{\hbar (m_f \Omega + m_b \omega_c)}.$$

As known, a physical state corresponds to a stable or metastable point of the energy functional. If a separation of the fermion and boson component occurs, then r_{fc} that minimizes the energy E_f takes a positive nonzero value.

In other words, there are no separations between the two components when energy E_f exhibits a minimal value at $r_f = 0$. When distribution function is restricted to the form of the trial function (15) we may write the conditions of a minimal energy in terms of derivatives of the energy with respect to the adjustable variation parameters of the trial function. We show them as follows

$$\begin{aligned} \frac{\partial E_f}{\partial \Omega}|_{\Omega=\Omega_c} = 0, \quad \frac{\partial E_f}{\partial r_f}|_{r_f=r_{fc}} = 0, \\ \frac{\partial^2 E_f}{\partial \Omega^2} \frac{\partial^2 E_f}{\partial r_f^2} - \left(\frac{\partial^2 E_f}{\partial \Omega \partial r_f} \right)^2 > 0, \end{aligned} \quad (17)$$

the stationary conditions (there are no separation between the boson and fermion) are

$$\begin{aligned} Y = Y(N_b, \omega_c, g_{bf}) = [\hbar \frac{\omega_f^2}{\Omega_c^3} + g_{bf} N_b \pi^{\frac{3}{2}} \sqrt{G(\Omega_c, \omega_c)} \frac{\partial^2 G(\Omega_c, \omega_c)}{\partial \Omega^2} + \frac{g_{bf} N_b \pi^{3/2}}{2G(\Omega_c, \omega_c)} \left(\frac{\partial G(\Omega_c, \omega_c)}{\partial \Omega} \right)^2] \\ \times [m_f \omega_f^2 - 2g_{bf} N_b \pi^{3/2} G^{5/2}(\Omega_c, \omega_c)] > 0, \end{aligned} \quad (18)$$

where Ω_c determined by $\partial E_f / \partial \Omega|_{\Omega_c} = 0$ satisfies the following equation

$$\frac{2}{3\pi} (6\pi^2)^{\frac{2}{3}} \left(\frac{3}{5} \right)^{\frac{3}{2}} \hbar N_f^{\frac{2}{3}} - \frac{1}{2} \hbar \left(\frac{\omega_f}{\Omega_c} \right)^2 + g_{bf} N_b \pi^{\frac{3}{2}} \sqrt{G(\Omega_c, \omega_c)} \frac{\partial G(\Omega_c, \omega_c)}{\partial \Omega} = 0, \quad (19)$$

the solution of Eq.(19) as a function of g_{bf} is shown in Fig.2, a magnification part of the curve near $g_{bf} = 0$ (but $g_{bf} > 0$) is give in the inset. This curve indicates that the fermions prefer to occupy the trap centre for $g_{bf} < 0$, and the larger the coupling constant $|g_{bf}|$ ($g_{bf} < 0$), the sharper the distribution of the fermions. However, as we show below, the fermions and the bosons can not always coexist even if $g_{bf} < 0$. $\frac{\partial E_f}{\partial r_f}|_{r_f=r_{fc}} = 0$ has two solutions, one solution is $r_{fc} = 0$ and the another is

$$r_{fc} = \sqrt{\frac{1}{G} \ln(2g_{bf} N_b \pi^{\frac{3}{2}} G^{\frac{5}{2}}(\Omega_c, \omega_c)) - \ln(m_f \omega_f^2)}. \quad (20)$$

For $g_{bf} < 0$ or

$$0 < g_{bf} < \frac{m_f \omega_f^2}{2N_b \pi^{\frac{3}{2}} G^{\frac{5}{2}}(\Omega_c, \omega_c)}$$

i.e. the interaction between fermion and the boson is attractive or weakly repulsive, the solution $r_{fc} = 0$ holds, which indicate that there is not separation between the fermions and the bosons. For $g_{bf} > \frac{m_f \omega_f^2}{2N_b \pi^{\frac{3}{2}} G^{\frac{5}{2}}(\Omega_c, \omega_c)}$ the fermions

experience a effective potential minimum at $r_{fc} = \sqrt{\frac{1}{G} \ln(2g_{bf} N_b \pi^{\frac{3}{2}} G^{\frac{5}{2}}(\Omega_c, \omega_c)) - \ln(m_f \omega_f^2)} > 0$, the Bose condensate is surrounded by a shell of fermions in this case. $Y(N_b, g_{bf}, \omega_c)$ as functions of the coupling constant g_{bf} are shown in figure 3. We see that whether the boson-fermion mixture is stable depends not only on the coupling constant g_{bf} and g_{bb} (through ω_c), but also on N_b and N_f (through Ω_c), i.e., the stability of the mixture system depends on the number of both boson and fermion system. For example, in Fig.3-a we show Y given by Eq.(8) as a function of the coupling constant g_{bf} for fixed $N_f = 100$, $N_b = 10000$, while Fig.3-b is for the same parameters as in Fig.3-a except for $N_b = 1000$, it is obvious that the region of g_{bf} in which the system has no phase separation has been broadened with N_b decreases (for fixed N_f). The inset present the dependence of Y on g_{bf} at a larger scale of g_{bf} . It is interesting to compare the above mentioned results with those obtained by treating the fermions in the Thomas-Fermi approximation, this is done in Ref.[13,14], and we note that the semiclassical description gives a qualitatively correct description and it reliably predicts the phase separation.

Now we tune our attention to discuss the above problem at finite temperature. First of all, we consider the homogenous case, for the boson and fermion system, thermodynamical properties are trivial if there are not interaction between them. But in this case the sympathetic cooling scheme does not take any effect and the degenerate fermions in a trapped potential have not been achieved. The thermodynamical properties may be changed when the interaction between the fermions and bosons is turn on, then a new phenomenon, the phase separation, may occur in this system. for a homogeneous fermion and boson mixture system, the Helmholtz free energy can be written as[16]

$$\beta F = -\frac{V}{\lambda_z^3} f_{\frac{5}{2}}(z_f) + \frac{1}{2} g_{ff} \rho_f N_f \lambda_f^2 + \ln(1 - z_b) - \frac{V}{\lambda_b^3} g_{\frac{5}{2}}(z_b) + 2g_{bb} \rho_b N_b \lambda_b^2 + g_{bf} (\lambda_b^2 + \lambda_f^2) N_f N_b / V, \quad (21)$$

where index f refers to the fermionic component, whereas index b stands for the bosonic one, N_i is the number of particles in component i , λ_i denotes the thermal wave length of component i , $f_n(z)$ and $g_n(z)$ represent the Fermi and Bose integral, respectively. The equation (21) is based on the pseudopotential form of the atom-atom interaction, and may be assumed accurate when the system is dilute. i.e. $\rho_i g_{ii}^3 \ll 1$ and $g_{ii}/\lambda_i \ll 1$, where ρ_i is the density of the component i . This condition is well satisfies for the samples of alkali atoms in experiments to date[1,12,17,18].

From eq.(21) we obtain the chemical potential for each component straightforwardly,

$$\begin{aligned}\beta\mu_b &= \beta\mu_b^0 + 4g_{bb}\rho_b\lambda_b^2 + g_{bf}(\lambda_b^2 + \lambda_f^2)N_f/V, \\ \beta\mu_f &= \beta\mu_f^0 + g_{ff}\rho_f\lambda_f^2 + g_{bf}(\lambda_b^2 + \lambda_f^2)N_b/V,\end{aligned}\tag{22}$$

where μ_i^0 are the chemical potentials of ideal gas. There are three terms in each chemical potential, the second term comes from the interaction within the component and the third term is from the interaction between the fermion and boson component. As known, an homogenous binary mixture is stable only when the symmetric matrix $\hat{\mu}$ given by

$$\hat{\mu} = \begin{bmatrix} \frac{\partial\mu_b}{\partial\rho_b} & \frac{\partial\mu_b}{\partial\rho_f} \\ \frac{\partial\mu_f}{\partial\rho_b} & \frac{\partial\mu_f}{\partial\rho_f} \end{bmatrix}\tag{23}$$

is non-negatively definite, in other words, all eigenvalues of matrix $\hat{\mu}$ given in Eq.(23) are non-negative. Mathematically, for homogeneous fermion and boson mixture the stability conditions are

$$\frac{\partial\mu_b}{\partial\rho_b} \geq 0, \frac{\partial\mu_f}{\partial\rho_f} \geq 0,\tag{24}$$

and

$$\det \begin{bmatrix} \frac{\partial\mu_b}{\partial\rho_b} & \frac{\partial\mu_b}{\partial\rho_f} \\ \frac{\partial\mu_f}{\partial\rho_b} & \frac{\partial\mu_f}{\partial\rho_f} \end{bmatrix} \geq 0.\tag{25}$$

For ideal gas, we have $\rho_b = \frac{1}{\lambda_b^3}g_{\frac{3}{2}}(z_b)$, $\rho_f = \frac{1}{\lambda_f^3}f_{\frac{3}{2}}(z_f)$, this leads to

$$\beta\frac{\partial\mu_f^0}{\partial\rho_f} = \frac{\lambda_f^3}{f_{\frac{1}{2}}(z_f)}, \beta\frac{\partial\mu_b^0}{\partial\rho_b} = \frac{\lambda_b^3}{g_{\frac{1}{2}}(z_b)},\tag{26}$$

It follows from eqs (24) and (25) that

$$4g_{bb}\lambda_b^2 + \frac{\lambda_b^3}{g_{\frac{1}{2}}(z_b)} \geq 0,\tag{27}$$

$$g_{ff}\lambda_f^2 + \frac{\lambda_f^3}{f_{\frac{1}{2}}(z_f)} \geq 0,\tag{28}$$

and

$$Z(T, g_{bf}, g_{ff}, g_{bb}) = Z = (4g_{bb}\lambda_b^2 + \frac{\lambda_b^3}{g_{\frac{1}{2}}(z_b)})(g_{ff}\lambda_f^2 + \frac{\lambda_f^3}{f_{\frac{1}{2}}(z_f)}) - g_{bf}^2(\lambda_b^2 + \lambda_f^2)^2 \geq 0.\tag{29}$$

It is well known that a homogeneous imperfect gas with attractive interaction is not stable. The fermions in this kind of gas could form BCS state, which consists two fermionic particles interacting with each other but not with the other fermions from the Fermi gas, whereas bosons with attractive interaction could collapse into liquid. Hence, we here discuss the system with repulsive interactions. It is obvious that the stability condition (27) and (28) hold always for $g_{bb} > 0$, $g_{ff} > 0$. We would like to point out that the stability conditions (27-29) do not involve the densities of the both components. At first sight, this seems to be confusion, in fact, there is no contradiction. One can demonstrate that at low density the Helmholtz free energy of the bogoliubov gas reduce to a quadratic form in N_b and N_f . To have a minimum, this form should be positive definite, i.e., $\det||\frac{\partial^2 F}{\partial N_b \partial N_f}|| \geq 0$. Therefore, the corresponding stability criterion involves only density-independent constants in the order of approximation used. This criterion is similar to the stability conditions for two-component Bose-Einstein condensate in a trapped untracold gas[19-25]. When $T \rightarrow \infty$, $\lambda_i \rightarrow 0$, hence $Z \sim \frac{1}{\rho_b \rho_f}$. Thus at high temperature, the homogeneous binary gas mixture is always stable

and no phase separation occur. In the case considered here, Fermi temperature $T_F = \frac{\hbar^2}{2mk_B}(\frac{3N_f}{8\pi V})^{\frac{2}{3}}$ is much lower than BEC temperature $T_c = \frac{\hbar^2}{2\pi mk_B}(\frac{N_b}{2.612V})^{\frac{2}{3}}$, i.e., as temperature decreases, it first passes the BEC transition point T_c . When $T \rightarrow T_c$, $g_{\frac{1}{2}}(1) \rightarrow \infty$, so

$$Z(T, g_{bf}, g_{bb}, g_{ff}) \sim 4g_{bb}\lambda_b^2(g_{ff}\lambda_f^2 + \frac{\lambda_f^3}{f_{\frac{1}{2}}(z_f)}) - g_{bf}^2(\lambda_b^2 + \lambda_f^2)^2.$$

In particular, when $T \ll T_F$, i.e., the temperature is much smaller than the Fermi temperature of the fermion system, the stability condition becomes (setting $m_f = m_b$)

$$g_{bb}g_{ff} - g_{bf}^2 \geq 0, \quad (30)$$

which does not depend on temperature and coincides with the stability conditions of two-component BEC[19,25]. Although it is difficult to reach this region of very low temperature, yet it attracts much more attention. Because both superfluidity and shell effects are expected to occur at temperature much smaller than the Fermi temperature[6,26]. Z given by Eq.(29) as a function of the temperature is shown in Fig.4, we see that the system is always stable when $T \rightarrow 0$ and $T \rightarrow \infty$, and the system is unstable for $T_{c1} < T < T_{c2}$, where T_{c1} and T_{c2} are roots of $Z(T, g_{bb}, g_{bf}, g_{ff}) = 0$. In particular, T_{c1} and T_{c2} depend on g_{bf} , g_{ff} and g_{bb} . As g_{bf} decreases (for fixed g_{bb} and g_{ff}), T_{c1} tends to T_{c2} (in Fig.4 going from dotted line to solid line). The critical temperature T_{c1} and T_{c2} characterize the onset of the phase separation, which is quite different from the Bose-Einstein condensation and the degenerate fermions. The critical temperature of BEC and of the onset of degenerate fermionic gas depend mainly on the density of the system N_i/V ($i = b, f$). Especially, the BEC and the degenerate fermionic gas may happen even if $g_{bf} = 0$. For the phase separation, however, nothing will happen if $g_{bf} = 0$. For a fixed temperature and the coupling constant g_{bf} , Z vs. g_{bb} and g_{ff} is shown in Fig.5, which represents the dependence of the stability on the interaction strength inside each component.

Until now, we considered only a homogeneous Fermi-Bose gas mixture at finite temperature. In reality, however, experiments with ultracold atoms are performed by trapping and cooling in an external potential that can be generally modeled by an isotropic harmonic oscillator $V(r) = \frac{m}{2}\omega_t^2 r^2$, where ω_t is the trapping frequency. An exact criterion for the stability of an inhomogeneous Bose-fermi mixture should involve calculating the Helmholtz free energy as a function at all eigenstates of the trapping potential. Fortunately, in the system considered here it is a good approximation to take use of the local-density approximation, which treats the system as being locally homogeneous. This requires that the level spacing $\hbar\omega_t$ of the trapping potential is much smaller than the Fermi energy. Of course, the local density approximation always breaks down at the edge of the gas cloud where the density vanishes and the effective Fermi energy becomes zero. In this approximation, the stability conditions can still be calculated by means of the equations derived above, with the understanding that now the effective chemical potentials are spatially dependent through

$$\mu_b = \mu_b^0 - \frac{1}{2}m_b\omega_b^2 r^2, \quad \mu_f = \mu_f^0 - \frac{1}{2}m_f\omega_f^2 r^2.$$

Thus a local stability condition is the same as given in Eq.(29) but replacing z_i ($i = 1, 2$) by

$$\tilde{z}_1 = z_1 e^{-\frac{\beta}{2}m_b\omega_b^2 r^2}, \quad \text{and} \quad \tilde{z}_2 = z_2 e^{-\frac{\beta}{2}m_f\omega_f^2 r^2}.$$

As shown in inset of Fig.4, the region of temperature in which the system is unstable decrease for the case of $r \neq 0$. As compared with the case without trapped potential, the total energy of the system increases for it in a trap. Alternatively, within the TFA, the chemical potentials decrease in this process. So this effect is equal to be that the particle number of the system has a loss. In this sense, the system is more stable than before.

In summary, we considered a dilute Bose-Fermi gas mixture in an isotropic trap. The atom can interact via s-wave scattering except within the fermions. These interactions strongly affect the stability of the system at zero and finite temperature. In addition, the stability conditions depend on the ratio rate N_b/N_f , the larger the ratio rate, the smaller the region of stability. For finite temperature, however, the stability conditions depends not only on the interactions, but also on the temperature. The region $T_{c1} \leq T \leq T_{c2}$ in which the system is unstable depend on the strength of the interaction between and within the bosons and the fermions. For an anisotropic trap, the stability conditions remain unchanged, whereas somewhat would be changed for zero temperature compared with the case of isotropic trap. To study the effects, we should introduced the another variation parameter in Eqs (9) and (5) to characterize the BEC and the fermions in this trap. Consequently, the stability condition (18) for zero temperature changes and the phase separation could different for different orientation. These need further investigations.

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Figure captions:

Fig. 1:The parameter ω_c which minimizes the energy functional versus the coupling constant g_{bb} . The trapped frequency $\omega_b = 166Hz$ and the number of the bosonic atom $N_b = 1000$.

Fig. 2:The parameter Ω_c which minimizes the energy functional as a function of the coupling constant g_{bf} . The parameters chosen are $g_{bb} = 0.05$ in units of $\hbar\omega_f a^3$ ($a = \sqrt{\hbar/\omega_b m_b}$) and all the coupling constants are chosen in this units hence forth, $N_f = 100$, $\omega_f = 166Hz$. Scatter and solid line correspond to different number of bosonic atom, as specified in the figure. The inset presents the enlarged part of the curve near $g_{bf} = 0(> 0)$.

Fig.3:Plot of Y given by Eq.(18) as a function of coupling constant g_{bf} . The parameter chosen are a: $N_f = 100$, $N_b = 1000$. b: $N_f = 100$, $N_b = 10000$. The curve for a larger scale of g_{bf} is presented as an inset in the figure.

Fig.4:Plot of Z given by Eq.(29) as a function of temperature T . The parameters chosen are $N_b = 1000$, $N_f = 10000$, $g_{bb} = 0.05$, $g_{ff} = 0.01$. Dashed-dotted line: $g_{bf} = 0.3$, dotted line $g_{bf} = 0.02$, solid line $g_{bf} = 0.01$. The dotted line in the inset is the same as the dotted line in the figure, while the solid line in the inset is for the gases in a trap with trapped frequency $166Hz$.

Fig.5:Plot of Z as a function of g_{ff} and g_{bb} . The parameters chosen are temperature $T = 0.1T_F$, $g_{bf} = 0.2$.